Project Title: The Dynamics of inequality

Decision making in stochastic processes

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Contents

Note: We both tried to capture the essence of the paper, and have tried to do justice to the paper by explaining it as well as we could. The summary of the paper may be a little more than 10 pages, but please do keep in mind that the paper we summarized was 40+ pages (and that the first 2 pages and last 2 pages are redundant)
Appendix C - I were not available in the paper, even though they were mentioned a number of times. We did our best in understanding the paper without those.

1 Introduction: Main motivation of Paper

Index terms (from class) *– Stochastic Differential Equation, Stochastic processes, Ergodicity / Ergodic Theory, Brownian Motion, Fokker-Plank Equation / Kolmogorov Forward equation, Random Growth Model, Jump Process / Poisson process, Quadratic Variation, Transition Matrix, Ito's lemma*

United states has seen a rapid growth in top inequality in the past forty years, i.e. the rate at which super rich are getting richer is more than the rate at which rich are getting richer, due to which the inequality between the two groups is increasing. The reasons for this is an open question. In the two deviations that this paper proposes, they consider two reasons namely, "changes in skill prices" or "rise in superstar entrepreneurs or managers".

As we know, wealth is distributed according to Pareto distribution which basically implies small number of customers are responsible for a very large sales turnover. Due to the rise in top inequality the tail of the pareto distribution shown is figure is becoming fatter.

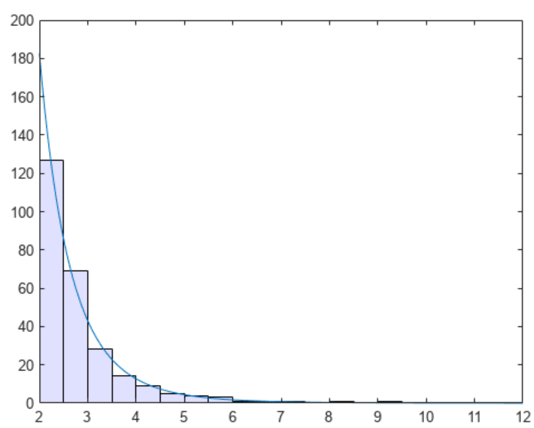


Figure 1: Example Pareto Distribution

The main contributions by the paper are:

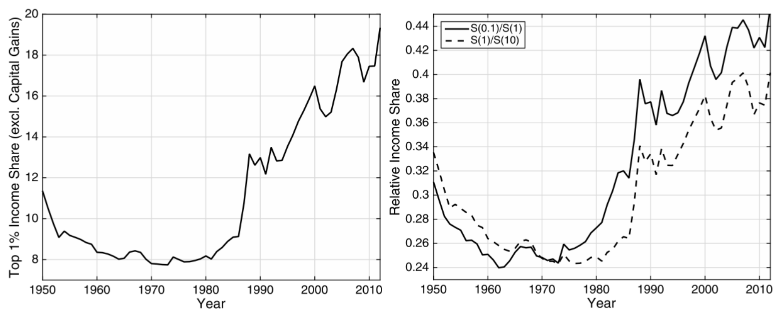
- They demonstrated that previous prevalent theories(Gibrat's formula income dynamics etc.) can't account for this sharp increase in top inequality.
- They suggest two parsimonious (Simple models with strong explanatory and predictive ability are called parsimonious models. They use the fewest possible parameters, or predictor variables, to describe the data) deviations from the basic models that can explain this sharp increase.

In figure below we can observe the fast rise in top inequality. The author's "augmented random growth model" differs from standard models by two minor deviations which explain quick transitions. Both deviations are from Gibrat's Law. Gibrat's law states that the distribution of income growth rates is independent of income level. Gibrat's law can also be stated as "the growth of the firm is independent of the firm size". The paper proposes two reasons for the increase in top-level inequality.

- **Type Dependence (of the growth rate distribution):**
  Due to presence of people who can be considered "High growth types", i.e. the authors refer to highly skilled managers/ entrepreneurs who will have substantially higher average salary growth rates than others over short or medium periods of time. It has

been analytically and quantitatively shown in the paper that this can explain the fast rise in top inequality.

- **Scale Dependence (of the growth rate distribution):**
  Due to shocks that disproportionately affect high-income individuals. Shocks that generate scale dependence affect log income multiplicatively, rather than additively, as in the usual random growth models. A great example of this is changes in skill pricing in an assignment model. Such changes are capable of producing quick shifts in inequality.



(a) Top Income Inequality (b) Relative Income Shares

FIGURE 1.—Evolution of top 1% income share and "fractal inequality" in U.S.

Figure 2: Motivation: Fast rise in top inequality

To find the analytical formulae for speed of convergence, they use the ideas of **ergodic theory** and theory of partial differential equations. As discussed in the class, ergodic theory is the study of study of the long term average behavior of systems evolving in time. We will not go a lot in details of the proof but we found the basic principles of deriving speed of convergence was done using ergodic theorems we learnt in class. The proof in Appendix 1 is based on ergodicity.

Stochastic Partial Differential Equations is an interdisciplinary area at the crossroads of stochastic processes (random fields) and partial differential equations. Generally speaking, any partial differential equation should be classified as an SPDE if its coefficients, forcing terms, initial and boundary conditions, or at least some of the above are random.

We also observe how Ito's lemma and Brownian motion concepts are being used to understand the fast changes in top inequality i.e. how these concepts are extremely useful in understanding real world systems. As discussed during this course, **Ito's lemma** is an identity used to find the differential of a time-dependent function of a stochastic process. It serves as the stochastic calculus counterpart of the chain rule. The lemma is widely employed in mathematical finance. On the other hand, **Brownian motion** is by far the most important stochastic process. It is the archetype of Gaussian processes, of continuous time martingales, and of Markov processes. It is basic to the study of stochastic differential equations, financial mathematics etc.

Through this paper, we try to understand the speed of convergence of the entire cross sectional distribution using general **stochastic processes**. This makes it easier for them to investigate inequlity dynamics.

2 Previous Literature: Random Growth Theories of Income Inequality

In this section, authors look at previous theories which have attempted to explain top income inequality.

2.1 Income Dynamics

Time is continuous, and there is a continuum of workers indexed by i. Workers are heterogeneous in their income/wage w_{it}. (Note: $x_{it} = log w_{it}$.) The reduced form of income dynamics:

$$dx_{it} = \mu dt + \sigma dZ_{it} + g_{it} dN_{it} \tag{1}$$

where Z_{it} is a standard Brownian motion and N_{it} is a jump process with intensity ϕ (like a Poisson jump process). (We can see how this is similar to what we learnt in class, in lectures 10-15.) The innovations g_{it} are drawn from an exogenous distribution f, and f can be either thin tailed or fat tailed.

To produce a stationary distribution, a stabilizing force is supplied, and if it is not provided, the variance of x_{it} has no bound. There are two stabilizing elements in this paper:

- Workers may retire/die at rate δ, after which they are replaced by fresh workers paid x_{it} from the distribution $\psi(x)$.
- Instead of taking 0 as the lower income, we give a lower income limit ($\underline{x}$). Employees resign if their earnings fall below a certain level. We assume $\underline{x} = 0$, then $\underline{w} = 1$. A reflecting barrier represents this lower bound. We think about the exit at $\underline{x}$ with the entrance (i.e., reinjection) at a point $x > \underline{x}$ taken from the distribution $\rho(x)$.

The technique described in (1) assumes that μ, σ, and f do not depend on income (Gibrat's law) and that the coefficients remain constant throughout time. These assumptions, according to the report, can be modified. And that drift and diffusion are arbitrary functions of the income level $\mu(x,t)$ and $\sigma(x,t)$ that converge to constants over time.

2.2 Stationary Income Distribution

The characteristics of a stationary distribution for the process described in (1) for the income logarithm are well known. In particular, this stable distribution features a Pareto tail when certain parameter constraints are fulfilled.

$$P(w_{it} > w) \sim Cw^{-\zeta} \implies P(x_{it} > x) \sim Cx^{-\zeta x}$$

where C is a constant, $\zeta > 0$ and is a function of parameters μ, σ and distribution of jumps f. When there are no jumps i.e., $\phi = 0$, ζ is the positive root of:

$$0 = \frac{\sigma^2}{2}\zeta^2 + \mu\zeta - \delta \tag{2}$$

Solving that we get, (ζ is a power law exponent)

$$\zeta = \frac{-\mu + \sqrt{\mu^2 + 2\rho^2\delta}}{\rho^2} \tag{3}$$

Smaller ζ corresponds to a fatter tail. The top inequality can be calculated as $\eta = 1/\zeta$.

It is unclear if the above two models can explain the fatter pareto tail behavior. i.e. observed dynamic of pareto tail parameter η.

3 The baseline random growth model generates slow transition

Previous research concentrated on steady distributions but not on transitions. Changes in the parameters of the revenue process (1) cause changes in the fatness of the right tail of the stationary distribution. The standard growth model (1) generates relatively sluggish transition dynamics. Basically, previous papers do say that the in pareto distribution the right tail is fatter but they do not talk about the speed at which the rise in inequality is occurring. Hence, the authors conduct the experiment: Initially at time $t = 0$, the economy is in Pareto steady state, with initial parameters: μ_0, σ_0^2 and then some parameter changes, we can assume the innovation variance σ^2 increases. Asymptotically at $t \to \infty$ the stationary distribution changes. The paper presents:

- the average speed of convergence through out the distribution
- the differential speeds of convergence across the distribution, which allows us to put spotlight on upper tail.

We denote the cross-sectional distribution of the logarithm of income x at time t by $p(x,t)$, the initial distribution by $p_0(x)$, and the stationary distribution by $p_\infty(x)$. They used the L1-norm to measure the distance between the distribution at time t and the stationary distribution.

$$\left\| p(x,t) - p_\infty(x) \right\| := \int_{-\infty}^{\infty} \left| p(x,t) - p_\infty(x) \right| dx \tag{4}$$

The cross sectional distribution obeys the Kolmogorov Forward Equation also taught to us as **Fokker-Plank equation** which *drift* $= \mu$, *Diffusion Coefficient* $= D = \frac{\sigma^2}{2}$. **Fokker-Plank equation** / Kolmogorov Forward Equation for probability density $p(x,t)$ is:

$$\frac{\partial}{\partial t} p(x,t) = -\frac{\partial}{\partial t}[\mu p(x,t)] + \frac{\partial^2}{\partial^2 t}[\frac{\sigma^2}{2} p(x,t)]$$

Using this we can get the cross sectional distribution *p(x,t)* without jumps:

$$\frac{\partial p(x,t)}{\partial t} = -\mu \frac{\partial p(x,t)}{\partial x} + \frac{\sigma^2}{2} \frac{\partial^2 p(x,t)}{\partial^2 x} - \delta p + \delta \psi$$

Which can also be written as:

$$p_t = -\mu p_x + \frac{\sigma^2}{2} p_{xx} - \delta p + \delta \psi \tag{5}$$

The first two terms on the right-hand side of (5) describe the development of x owing to diffusion, with drift μ and variance σ^2. The third term represents death, i.e., an outflow of humans at rate δ. The fourth phrase catches birth, i.e., every dying/retiring individual is replaced by a new born pulled from $\psi(x)$.

When there is a reflecting barrier, p must satisfy the boundary condition (which was explained on top):

$$0 = -\mu p + \frac{\sigma^2}{2} p_x \quad \text{at} \quad x = 0 \quad \forall t \tag{6}$$

When there is exit at x = 0, with reinjection at points greater than x = 0, $\rho = 0$, the boundary condition is: $p(0,t) = 0 \quad \forall t$

The above boundary condition comes from integrating (5) from x = 0 to x = ∞, and we get:

$$0 = \mu p(0,t) - \frac{\sigma^2}{2} p_x(0,t) + \gamma \quad ; \quad \gamma = \frac{\sigma^2}{2} p_x(0,t) \qquad \text{(This captures the reinjection after exit)}$$

After taking the reinjection into account, the equation becomes: $p_t = -\mu p_x + \frac{\sigma^2}{2} p_{xx} - \delta p + \delta\psi + \gamma p$

When there are jumps ($\phi \neq 0$), the **Fokker-Plank equation** is written as:

$$p_t = -\mu p_x + \frac{\sigma^2}{2} p_{xx} - \delta p + \delta\psi + \phi\mathbb{E}[p(x-g) - p(x)] \tag{7}$$

Note: A g leap at *(x - g)* transports *p(x - g)* persons to position x, therefore the name $\phi\mathbb{E}[p(x-g)$. $\phi p(x)$ persons leave place x when they jump at x, thus the phrase - $\phi p(x)$. The net result is $\phi\mathbb{E}[p(x-g) - p(x)]$. Since it's more understandable, the partial differential equations are written in terms of a differential operator. The equation (5) is written as:

$$p_t = A^* p + \delta\psi \quad \text{Where} \quad A^* = -\mu p_x + \frac{\sigma^2}{2} p_x x - \delta p \tag{8}$$

3.1 Average Speed of Convergence

The authors present a few assumptions and propositions in this part that are utilized frequently. Some statements are proved intuitively, while others serve as the foundation for further propositions.

ASSUMPTION 1:
The initial distribution $p_0(x)$ satisfies $\int_{-\infty}^{\infty} \frac{(p_0(x))^2}{e^{-\zeta x}} dx < \infty$ where $\overline{\zeta} := \frac{-2\mu}{\sigma^2} \leq \zeta$, and μ, σ are the parameters of the new steady-state process.
Proof: We know that in steady state, the income distribution has a Pareto tail. Using equation (2) and assuming that $\delta = 0$ i.e., there are no deaths/retirements, we get

$$0 = \frac{\sigma^2}{2}\zeta^2 + \mu\zeta \quad \rightarrow \quad \zeta(\frac{\sigma^2}{2}\zeta + \mu) = 0$$

$$\zeta = 0 \quad \text{or} \quad \zeta = \frac{-2\mu}{\sigma^2} \quad \text{Therefore} \quad \overline{\zeta} = \frac{-2\mu}{\sigma^2} \leq \zeta$$

From Assumption 1, we can then say that $\zeta_0 > \frac{\overline{\zeta}}{2} \rightarrow \zeta_0 > \frac{\zeta}{2}$, and in terms of top inequality, $\eta = \frac{1}{\zeta} : \eta_0 < 2\eta$.
This assumption allows us to rule out scenarios in which the initial steady-state top inequality is more than twice as great as the new steady-state top inequality. It is satisfied in particular when the top inequality in the new steady state is greater than the top inequality in the first steady state.

PROPOSITION 1 - AVERAGE SPEED OF CONVERGENCE:
Consider the income process (1) with death and/or a reflecting barrier as a stabilizing force but without jumps ($\phi = 0$). The cross-sectional distribution p(x,t) converges to it's stationary distribution $p_\infty(x)$ in the total variation norm for any initial distribution $p_0(x)$. The rate of convergence:

$$\lambda := -\lim_{t \to +\infty} \frac{1}{t} \left\| p(x,t) - p_\infty(x) \right\| \tag{9}$$

depends on whether there is a reflecting barrier at x = 0. Without a reflecting barrier,

$$\lambda = \delta \tag{10}$$

Because people do not return to work, the speed of convergence is solely determined by death or retirement. *With a reflecting barrier, under Assumption 1, and for generic initial conditions:* The greater the value of δ, the more churning in the cross-sectional distribution and the faster the distribution settles down to its invariant distribution. $\delta = 0$ when reflecting barrier $\mu = 0$ and no death. According to (3), the stationary tail inequality in this situation is $\eta = \frac{1}{\zeta} = \frac{-\sigma^2}{2\mu}$, and so the speed of convergence may also be represented as:

$$\lambda = \frac{\sigma^2}{8\eta^2} = \frac{\sigma^2}{8(\frac{-\sigma^2}{2\mu})^2} = \frac{\mu^2}{2\sigma^2} \tag{11}$$

$$\lambda = \frac{\mu^2}{2\sigma^2}\mathbf{1}_{\{\mu<0\}} + \delta \tag{12}$$

Where $\mathbf{1}_{\{.\}}$ *is the indicator function* [Note: This also proves the same point that is stated in Proposition 2.] This assists us in proving the equation (12). Because equation (11), expects $\delta = 0$, we may remove this assumption and add δ to (11) to get (12).
This equation makes obvious sense. It asserts that the faster the transition, the bigger the standard deviation of growth rates rho and the lower the tail inequality eta; that is, high inequality and sluggish transition go hand in hand.

As previously stated, process (1) is rather limited since it implies that Gibrat's law applies everywhere in the state space. In reality, this assumption may be relaxed while still obtaining an upper bound on the speed of convergence. Consider the more generic process y in this regard.

$$dx_{it} = \mu(x_{it}, t)dt + \sigma(x_{it}, t)dZ_{it} \tag{13}$$

The growth and the standard deviation of income both depend on the state itself and time. We can also see that as income x becomes very large, the functions $\mu(x_{it}, t), \sigma(x_{it}, t)$ converge to a strict random growth process. [Note: Assumption 2 and Proposition 2 are not used, and hence not stated here.]

The operator A^* summarizes the complete dynamics of the x_{it} process (8). This operator generalizes a transition matrix for a finite-state process to processes having a continuum of states, such as (1). The key feature of A^* that governs the speed of convergence of p is its biggest eigenvalue, which is the second eigenvalue. The first principal eigenvalue of the transition matrix is **zero** and corresponds to the stationary distribution $p_\infty(x)$; however, the second eigenvalue governs the speed of convergence to this stationary distribution because the loadings of the initial distribution decay more quickly on all other eigenvectors.

The key contribution of Proposition 1 is to obtain an explicit formula for the 2nd eigenvalue of A^*.

3.2 Speed of Convergence in the Tail

- They start by looking into the potential that different portions of the distribution may converge at various rates.
- Establish that the higher tail of convergence is slower.
- Jumps are valuable for describing some aspects of the data, but they don't speed up the convergence of the cross sectional income distribution.

3.2.1 Speed of Convergence in the Tail without a Lower Bound on Income

Without a lower bound on income, the entire time path of the solution to **Fokker-Plank Equation** can be characterized in terms of the Laplace transform of p:

$$\hat{p}(\zeta, t) := \int_{-\infty}^{\infty} e^{-\zeta x} p(x, t)dx = \mathbb{E}[e^{-\zeta x_{it}}] \tag{14}$$

For $\zeta \leq 0$, the $-\zeta$th moment of the income distribution is naturally interpreted by the Laplace transform. They demonstrate that for any t, we can construct a clear analytic formula for the full time path of this object. We may trace out the behavior for different regions of the distribution by modifying the variable zeta (the more negative zeta, the more we know about the distribution's tail behavior).

3.2.2 Speed of Convergence in the Tail with reflecting Barrier

PROPOSITION 4:
Consider the income process (1) without jumps $\phi = 0$ but with a reflecting barrier. Under Assumption 1, the rate of convergence $\lambda(\zeta) := -\lim_{t\to+\infty} \frac{1}{t}\left\|p(x,t) - p_\infty(x)\right\|_\zeta$ of the weighted L1 norm (15) is:

$$\lambda(\zeta) = \begin{cases} \frac{1}{2}\frac{\mu^2}{\sigma^2} + \delta & \text{if } \zeta \geq \frac{\mu}{\sigma^2} \\ \mu\zeta - \frac{\sigma^2}{2}\zeta^2 + \delta & \text{if } \zeta < \frac{\mu}{\sigma^2} \end{cases} \tag{15}$$

3.3 The baseline Model Cannot Explain the Fast Rise in Income Inequality

The authors show that a simple change in σ cannot explain the rapid change in top income inequality as the transition dynamics of standard random growth model is very slow. The income of any individual depends on the economy, their position, etc.

4 Models that generate Fast Transitions

In this section we try to explain the cause of fast rise in top inequality. First augmented random growth model is explained. Then the extension of Gibrat's law by considering Type dependence and Scale dependence has been described. Finally it has been shown how the new augmented model is able to generate fast transitions observed in data.

4.1 The Augmented Random Growth Model

A general model for income including both type and scale dependence is given below:

$$\begin{aligned} x_{it} &= \chi_t^{b_j} y_{it}, \\ dy_{it} &= \mu_j dt + \sigma_j dZ_{it} + g_{jit} dN_{jit} + Injection - Death \end{aligned} \tag{16}$$

Here, type dependence is indexed by $j = 1, ...J$. χ_t captures scale dependency, which is an arbitrary stochastic process fulfilling χ_t and $\lim_{t\to\infty} \mathbb{E}[\log \chi_t] < \infty$. Basically, if $b_j > 0$, a rise in χ_t suggests that income growth is greater for those with higher incomes, which violates Gibrat's law. dN_{jit} is a Poisson process with intensity ϕ. g_{jit} is a random variable with distribution f_j. y_{it} is the worker's skill. Workers retire at rate δ, and get replaced by new workers with income drawn from ψ distribution.

4.2 The role of Type Dependence

- They exclusively evaluate the role of Type Dependence using equation (17), ignoring Scale Dependence ($\chi_t = 1$).
- They also presume there are two basic cases: high growth and low growth. Low growth is an absorbing state that is only left behind when one retires.

- Some people begin their careers as high-growth types, while others begin as low-growth types; people flip from high to low growth with some intensity α.
- Density of individuals in high-growth states: $p^H(x,t)$
  Density of individuals in low-growth states: $p^L(x,t)$
  Cross-sectional wage distribution: $p(x,t) = p^H(x,t) + p^L(x,t)$
- Fraction θ of individuals start their career as high-growth type, and the remainder as low-growth type.

The densities for the cross sectional wage follow the **Fokker-Plank equation**:

$$\begin{aligned} p_t^H &= -\mu p_x^H + \frac{\sigma_H^2}{2} p_{xx}^H - \alpha p^H - \delta p^H + \beta_H \delta_0 \\ p_t^L &= -\mu p_x^L + \frac{\sigma_L^2}{2} p_{xx}^L + \alpha p^H - \delta p^L + \beta_L \delta_0 \end{aligned} \tag{17}$$

The initial conditions are:

- $p^H(x,0) = p_0^H(x)$, $p^L(x,0) = p_0^L(x)$
- δ_0 is the Dirac delta function at x = 0 which captures rebirth
- The birth rates for the high and low growth types respectively: $\beta_H = \theta\delta, \quad \beta_L = (1-\theta)\delta$

Since, it's hard to solve the **Fokker-Plank equation** analytically, they used Laplace transform like we showed before and devised the below equations:

$$\begin{aligned} \hat{p_t^H}(\zeta,t) &= -\lambda_H \hat{p^H}(\zeta,t) + \beta_H \\ \lambda_H(\zeta) &:= \zeta\mu_H - \zeta^2 \frac{\sigma_H^2}{2} + \alpha_H \delta \end{aligned} \tag{18}$$

$$\begin{aligned} \hat{p_t^L}(\zeta,t) = -\lambda_L \hat{p^L}(\zeta,t) + \alpha \hat{p_H}(\zeta,t) + \beta_L \\ \lambda_L(\zeta) := \zeta\mu_L - \zeta^2 \frac{\sigma_L^2}{2} + \delta \end{aligned} \tag{19}$$

[Note: For fixed ζ, it's an ordinary differential equation, just like we did before]

PROPOSITION 5 - SPEED OF CONVERGENCE WITH TYPE DEPENDENCE: Consider the cross section distribution p(x,t) with stationary distribution also defined similarly, and that the stationary distribution has a Pareto tail with tail exponent $\zeta = min\{\zeta_L, \zeta_H\}$, where

ζ_H is the positive root of $0 = \zeta^2 \frac{\sigma_H^2}{2} + \zeta\mu_H - \alpha - \delta$

$$\zeta = \alpha + \delta \quad \text{or} \quad \frac{2 * (\alpha + \delta - \mu_H)}{\sigma_H^2}$$

ζ_L is the positive root of $0 = \zeta^2 \frac{\sigma_L^2}{2} + \zeta\mu_L - \delta$

$$\zeta = \delta \quad \text{or} \quad \frac{2 * (\delta - \mu_H)}{\sigma_L^2}$$

The transition dynamics are mentioned in the paper and will not be rewritten here.
The conclusion: The transition dynamics of the income distribution occur on two separate time scales: one at rate $\lambda_H(\zeta)$ and another at rate $\lambda_L(\zeta)$. The model has the theoretical capability of explaining short-run dynamics as well as the observed growth in income disparity.

4.3 The role of Scale Dependence

Consider the case of scale dependence $d\log\chi_t \neq 0$, and without type dependence ($J = 1$, i.e., only 1 growth type). The authors show when the "scope of CEO Talent" γ_t increases, the talent multiplier χ_t increases. This makes sense intuitively, as when the CEO is able to handle more people, it means he has grown and there is a growth in his ability. Since, each individual is different, the term y_{it} is dynamic! As we can see, this situation is so close to the present day micro-foundation of scale dependence and the economy.

Income is $w_{it} = (e^{y_{it}})^{\chi_t}$, so increase in χ_t leads to the flattening of the tail of the income distribution.

PROPOSITION 6 - INFINITELY FAST ADJUSTMENT IN MODELS WITH SCALE DEPENDENCE:

Consider (16) $x_{it} = \chi_t y_{it}$, where the distribution of y_{it} is stationary and where χ_t is an aggregate shock. This process adjusts at an infinitely fast rate: $\lambda = \infty$. Denoting by ζ_t^x and ζ_t^y the power law exponents of log income and skill x_{it} and y_{it}, we have $\zeta_t^x = \frac{\zeta^y}{\chi_t}$. The proof is simple manipulations in mathematics.

The process is exceedingly quick, as seen by the instantaneous alterations in the power law exponent. As a result, these rapid changes are compatible with theories in which rising top inequality is caused by shifting talent prices.

They also indicate that, even if the market experiences shocks over time, CEOs will continue to earn a lot, and the income discrepancy will widen. Returns to education are becoming more "convex" with time, which is consistent with scale dependency caused by shifting skill pricing. The experimentally supported source of quick transition is scale dependency.

4.4 Fast Transitions in the Augmented Model

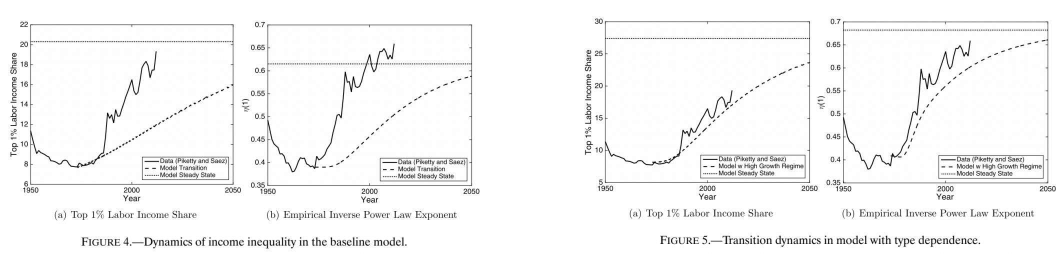


Figure 3: Comparison between baseline model and Type Dependence based model

It can be seen by the above figure that the baseline model does not follow the rise in top inequality. However, model with type dependence can replicate the rapid rise in income inequality observed in the United States very closely. From the above plots and quantitative exploration in previous sections we can say that model with type dependence can generate fast transition dynamics of top inequality for a number of alternative parameterizations. Whats common in these parameterizations is high growth rates(high μ_H) for a part of population and relatively short time periods (high α). Although, due to the lack of better microestimates for these crucial parameters, as well as the stylized character of our model, the quantitative investigations in this part should be regarded as exploratory.

5 Relevant Examples

Why do some individuals have money while others don't? How much power do governments have to influence inequality? What tools should they employ? Understanding why individuals save is necessary in order to respond to these queries. Dynamic quantitative models of wealth inequality can assist us in understanding and quantifying the factors that influence the results we see in the data and in assessing the effects of policy change.[1]

Another paper focuses on the fact that the bigger the firm size, the higher the salary of the CEO, which leads to top income inequality[2]. An interesting paper that we found was how stem and non-stem graduates affect the economic inequality. They compared this on individual, firm and country level. On individual level it was proven that STEM graduates contribute more to wealth inequality using **Zipf exponent**. In case of firms and countries a similar hypothesis has been tested and proven.[3] A very interesting paper studies the relationship between CEO payscale and trade openness. The results of the same show that trade openness impacts inequality within the very top of the income distribution, where skill differentials are less evident. This was a very unique approach to what we understood in the paper at hand. [4] This 2022 paper also tried to explain the rise in top inequality and similar to type dependence they put forth a model which considers either businessmen or workers. The decline in the interest rate increases wealth inequality since entrepreneurs benefit from lower financing costs while workers face lower returns. This channel can account for over 60 percent of the increase in the top wealth shares in the data. [5] All the papers mentioned have used the findings in our reference paper to establish more results on rise in top inequality and other topics. The paper has set the baseline results for future research.

6 Work Distribution

Ananya:

- *Introduction:* Understood the main motivation i.e., understanding why it's important to analyse the top income inequality, what is Pareto distribution, Gibrat's law and how they all work together in the paper. Tried finding the similarity in what we learnt in class and in the paper.
- *Previous Literature:* Understood the Stochastic differential equation and how the structure is similar to the Brownian motion models we learnt in class.
- *Baseline random growth model:* Understood the assumptions and propositions, how they were derived, why they are important and why they will not be able to explain the top income inequality gap. Understood the Gibrat's law and how it motivated this section in the paper. Worked out derivations of the Fokker-Plank equations which were inspired from Gibrat's law, and understood why they don't capture inequality.
- *Models that generate Fast Transitions:* Understood the main motivation behind the augmented models, and how each part has very significant contributions to the top income inequality. Worked on the derivations for the various augmented models. How type dependence, scale dependence are parsimonious derivations, and how they have actually captured the true working of the top income inequality.
- *Relevant Examples:* Read through a couple of papers which cited "Dynamics of Inequality". I tried to understand how this paper helped them to prove their hypothesis! Also tried understanding how this paper was used as a baseline for many other models which were then used to capture the top income inequality in US, and for understanding the financial working.
- Worked on the report, as well as the equations and derivations for the augmented growth models i.e., Attempted to solve the differential equations in both Sections 3 and 4.
- I attempted to relate the class literature to what we had learned and use that as motivation to comprehend this paper. Also tried to structure the document in a way which is easy to read.

Kimaya

- *Introduction:* Understood and explained main motivation behind the paper, its significance, what previous theories say, how that is not explaining the fatter pareto tail and what the paper is trying to prove i.e. the open question of why there is fast rise top inequality rises and what the two deviations are.
- *Previous Literature: Understood Gibrat's law, SPDE and related models mentioned in literature survey and similarity to Brownian motion learnt in lec10-15 in class. Partially understood why whether previous models explain the pareto tail behavior is unclear.*
- *Baseline random growth model:* Understood the assumptions behind the propositions and how how standard random growth model causes very slow transition dynamics, perceived the need for a new model as the previous models do not describe the rise in top inequality.
- *Models that generate Fast Transitions:* Attempted to understand how augmented random growth model works, role of type dependence and scale dependence
- *Relevant Examples:* Went through recent papers to see how the research in given paper has affected recent theories. It was very interesting to explore effect of STEM and non-STEM personal on top inequality, relationship between CEO payscales and openness in trading, also relationship between firm size and CEO payscale , Dynamic quantitative models of wealth inequality etc.
- Attempted to write everything in the report in simple terms so anyone without prior knowledge will be able to understand what is being said.
- Tried to understand which concepts from class were used in the literature
- Found literature to see how top income inequality has been explained by recent papers and also what other models have been given for explaining top inequality apart from the ones mentioned in our paper.

7 References

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3. Boris Podobnik, Marina Dabić, Dorian Wild, Tiziana Di Matteo, The impact of STEM on the growth of wealth at varying scales, ranging from individuals to firms and countries: The performance of STEM firms during the pandemic across different markets, Technology in Society, Volume 72, 2023, 102148, ISSN 0160-791X, `https://doi.org/10.1016/j.techsoc.2022.102148`.
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